Dynamics of random dipoles : chaos *vs* ferromagnetism

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Abstract. The microcanonical dynamics of an ensemble of random magnetic dipoles in a needle has been investigated. Due to the presence of a constant of motion in the 1–D case, a "dimensional" phase transition in the quasi one dimensional case has been found separating a paramagnetic chaotic phase from a ferromagnetic regular one. In particular, a simple criterium for the transition has been formulated and an intensive critical parameter found. Numerical simulations support our understanding of this complex phenomenon.

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1. Introduction

A truly comprehensive understanding of magnetism at the nanoscale is still lacking and has important consequences in the technology of memory and information processing devices.

Many unsolved problems about magnetic properties of diluted spin systems attracted recently great attention. Among the open problems there is the emergence of ferromagnetism in doped diluted systems[1], where the Curie temperatures can be as high as 300 K, and a deep theoretical understanding of the magnetic properties of dilute dipole systems (spin glass transition, ferromagnetic and anti-ferromagnetic transitions).

Here we will concentrate on randomly arranged dilute classical dipoles, which are called dipole glasses. Many results in literature, sometimes controversial, exist on such kind of systems. Magnetic properties of dipole-dipole interacting spins are particularly difficult to study due to many factors: long range nature of the interaction, anisotropy and frustration. Long range and anisotropy can induce ergodicity breaking[2] in a system. Breaking of ergodicity, a concept introduced by Palmer[2], and recently found explicitly [3, 4] in a class of long-ranged anisotropic spin systems, is a key word to understand phase transitions too, even if it should not be confused with breaking of symmetry[5]. Speaking loosely, few constants of motion, such as the energy, or the angular momentum, in a particular geometry, produce a separation of the allowable phase space in two or more subspace over which the motion is constrained. In Ref.[3] the energy at which the separation occurs has been calculated explicitly for an anisotropic 1–D classical Heisenberg systems. In that case both the anisotropy and the long ranged nature[6] of the inter-spin interaction, are essential ingredients in order to have breaking of ergodicity[7]. On the other hand, frustration, that is the impossibility to attain a global minimal energy minimizing locally the interactions, induces a dependence of the ferromagnetic and anti-ferromagnetic properties on the lattice geometry [8].

Other results concerning the so-called Ising dipole glass can also be found in literature, where Ising simply means uni-axial. To quote but a few: spin glass transition for high concentration, using Monte Carlo simulation[9, 10], mean field spin glass transition at low concentration depending on the lattice geometry[11], absence of spin glass transition for low concentration using Wang-Landau Monte Carlo simulations [12] or the recent spin glass transition at non zero temperature from extensive numerical simulation[13].

In this paper we will focus our analysis on a dipole glass of freely rotating classical dipoles. First of all the dipole glass is a typical example of very frustrated system [14, 15, 16], so that different ground state configurations can exist depending on the geometry and the spin concentration. Results in the canonical ensemble typically consider a mean field approach, and it is common lore that the random positions of the dipoles induce magnetic field fluctuations. These fluctuations do not vanish at $T \to 0$, unlike thermal fluctuations, and tend to suppress magnetic order even at $T = 0$[15, 16]. So, magnetic order, is expected to happen only for high impurity

concentration (and small temperature) [16, 17, 18]. Mean field theories consider only the equilibrium properties and do not take into account the time needed to reach the equilibrium situation and finite size effects. On the other hand the question of how long a metastable state can last is a major issue in determining the magnetic properties of a system.

In this paper we study the microcanonical dynamics, reserving the study of the influence of a thermal bath for further investigations. We analyze the microcanonical dynamics of dipoles put at the vertexes of a cubic lattice (so that their relative distance cannot be smaller than the lattice size), only on the basis of the Landau-Lifshitz-Gilbert equations of motion. 3–D dipole-dipole interacting systems can be realized quite easily in laboratory, for instance doping a non magnetic media with paramagnetic ions, weakly interacting with the lattice and with an inter-dipole distance sufficiently large in order to neglect Heisenberg interaction (low concentration). The choice of studying a random glass instead of a system composed of dipoles regularly arranged in some lattice is twofold: from one hand is it relatively easy to dope a system putting some paramagnetic doping ions in a random way inside any non-magnetic media. On the other hand, a 3-D cubic lattice with a full concentration of dopant ions $\delta = 1$, even if thin, does not have a ferromagnetic ground state [19], so that another type of transition should be considered (paramagnetic/antiferromagnetic) .

Anticipating some of the results, we have found that taking into account a typical experimental situation with needle-shaped sample, at low dipole concentration a further constant of motion appears that induce a kind of "phase" transition related to invariant tori, which separate the allowable phase space in many disconnected regions. This result seems to indicate that at very low concentration a system of random dipoles in a needle resemble a one dimensional arrangement of dipoles, and thus can have ferromagnetic behavior. In this particular case, the ergodicity breaking is not due to an increase of energy, but to an increase of perturbation, which means the tendency to a transformation from a needle shape (quasi 1–D system) to a cubic shape (3–D shape). In a sense, these results are more akin to the standard perturbation theory in classical dynamical systems[20, 21], re-interpreted in the light of phase transitions induced by demagnetization times [22].

In the future we are going to study the same system in contact with a thermal bath. In this case the presence of the ergodicity breaking found in [3, 4] should influence the demagnetization times. In the microcanonical case, the presence of this ergodicity breaking is hidden by the quasi-integrability of motion.

2. The Classical Model and the Perturbative Approach

Let us consider a system of N classical dipoles $\vec{\mu}_i$ randomly put at the nodes of a 3–D gridded box $R \times R \times L$, with $L \gg R$, and low concentration $\delta \ll 1$, as indicated in Fig. 1,

From the physical point of view it represents a dilute system of paramagnetic ions in a

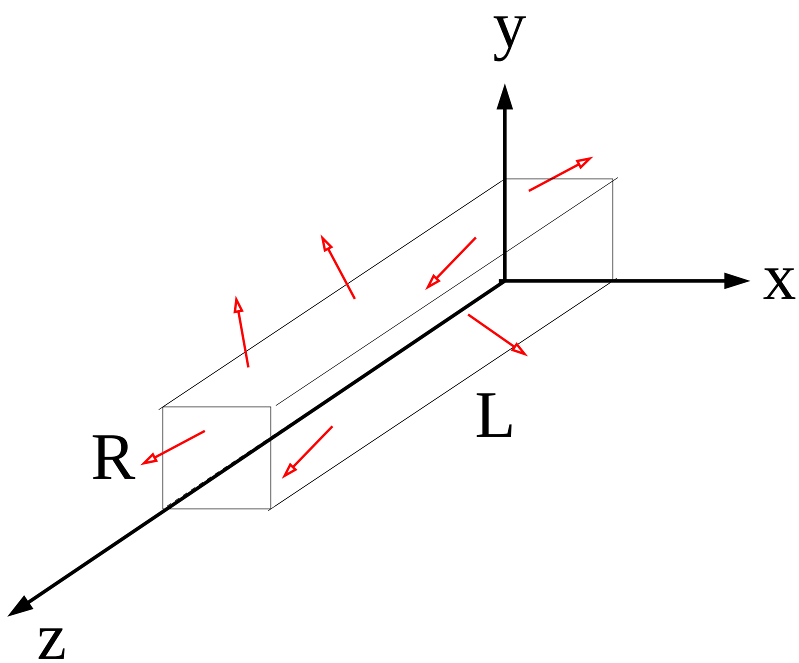

Figure 1. Needle geometry. The classical dipoles are put in a random way on the vertexes of a cubic lattice of size a. R and L are given in units of the lattice size a.

non magnetic bulk, with a concentration $\delta = N/N_s$ where $N_s = R^2 L$ is the number of allowable sites in the 3–D lattice. As explained above, such a system can be realized in laboratory, doping a non–magnetic system having a cubic lattice with paramagnetic impurities (e.g. doped TiO_2 and other[1]). If the dipoles weakly interact with the lattice and if their average distance is much greater than the Bohr radius, we can simply neglect the Heisenberg (exchange) interaction and represent their mutual interaction and dynamics with a pure dipole-dipole interaction energy:

$$E = \frac{\mu_0 \mu^2}{4\pi a^3} \sum_{i=1}^{N} \sum_{j>i} \frac{1}{|r_{ij}|^3} \left[\vec{S}_i \cdot \vec{S}_j - 3(\vec{S}_i \cdot \hat{r}_{ij})(\vec{S}_j \cdot \hat{r}_{ij}) \right]. \tag{1}$$

Here $\vec{S}_i$ is the i–th dimensionless spin vector

$$\vec{S}_i \cdot \vec{S}_i = 1, \tag{2}$$

μ is the magnetic moment of the paramagnetic doping ions and r_{ij} is the distance between the i-th and the j-th spin in units of the lattice spacing a.

The dynamics is described by the Landau-Lifshitz-Gilbert equations of motion:

$$\frac{d}{dt}\vec{\mu}_k = \gamma \vec{\mu}_k \times \frac{\delta E}{\delta \vec{\mu}_k}, \tag{3}$$

where $\vec{\mu}_k = \mu \vec{S}_k$ and γ is the gyromagnetic ratio.
They can be rewritten in the dimensionless form,

$$\frac{d}{d\tau}\vec{S}_k = \vec{S}_k \times \frac{\delta E_0}{\delta \vec{S}_k}, \tag{4}$$

where the following dimensionless quantities have been introduced:

$$\begin{aligned} E_0 &= E\frac{4\pi a^3}{\mu_0\mu^2} \\ \tau &= \omega t, \quad \text{with} \quad \omega = \frac{\gamma\mu\mu_0}{4\pi a^3}. \end{aligned} \tag{5}$$

The system of equations (4) conserves the energy (1) and the squared modulii of the spins (2).

The diluted quasi 1–D system can be magnetized with a strong magnetic field directed along L, the longest axis (z-axis). The questions we would like to answer is the following: What is the dependence of the average demagnetization time and its fluctuations on the system parameters?

The relevant parameters to take into account are the concentration δ of paramagnetic ions and the aspect ratio $\epsilon = R/L$. In principle, due to the long-ranged nature of the dipole interaction, one could ask whether there are effects dependent on both the system size and the number of doping spins N, even if in quasi 1–D systems, the dipole interaction can be treated as a short range interaction.

From the point of view of the equations of motion (4), if the N dipoles are lying along a straight line ($R = 0 \Rightarrow \epsilon = 0$), there is a further constant of motion, i.e. $M_z = (1/N)\sum_k S_k^z$. Therefore, for a 1–D system, the answer to the first question above is very simple : a state with any initial magnetization $M_z(0) \neq 0$ will keep the initial magnetization forever. The natural question thus becomes: what happens for $\epsilon \neq 0$? Will a magnetized state demagnetize and how much time it takes to do that?

The classical dynamical picture can be simplified adopting a perturbative approach, namely approximating the unit versor between two spins as follows:

$$\hat{r}_{ij} = \cos\theta_{ij}\hat{z} + \sin\theta_{ij}(\cos\phi_{ij}\hat{x} + \sin\phi_{ij}\hat{y}) \simeq \hat{z} + (\epsilon N)(\cos\phi_{ij}\hat{x} + \sin\phi_{ij}\hat{y}) \tag{6}$$

where $\hat{x}, \hat{y}, \hat{z}$ are the unit versors, ϕ_{ij} are the azimuthal angles with respect the z-axis and $\theta_{i,j}$ are the polar angles. In the last equation we approximate $\cos\theta_{ij} \simeq 1$ and $\sin\theta_{i,j} \simeq R/\langle d\rangle$, where, for dilute dipoles in a needle goemetry, $\langle d\rangle \simeq L/N$ is the average distance among spins. The energy (1), to first order in ϵN, becomes: $E_0 = H_0 + \epsilon N V$, where H_0 is the energy part that conserve M_z, and V is the perturbation,

$$\begin{aligned} H_0 &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}\frac{1}{|r_{ij}|^3}\left[S_i^x S_j^x + S_i^y S_j^y - 2S_i^z S_j^z,\right] \\ V &= -3\sum_{i=1}^{N}\sum_{j\neq i}\frac{1}{|r_{ij}|^3}\left[\cos\phi_{ij}S_i^z S_j^x + \sin\phi_{ij}S_i^z S_j^y\right]. \end{aligned} \tag{7}$$

The equations of motion for the macroscopic variables, $M_{x,y,z}$ can be written as,

$$\frac{dM_z}{d\tau} = 3\epsilon\sum_{i\neq k}\frac{1}{|r_{ik}|^3}S_i^z\left(S_k^y\cos\phi_{ik} - S_k^x\sin\phi_{ik}\right) \tag{8}$$

$$\frac{dM_y}{d\tau} = \frac{3}{N}\sum_{i\neq k}\frac{1}{|r_{ik}|^3}\left\{S_i^z S_k^x + \epsilon N\left[S_k^x S_i^y\sin\phi_{ik} + (S_k^x S_i^x - S_k^z S_i^z)\cos\phi_{ik}\right]\right\}$$

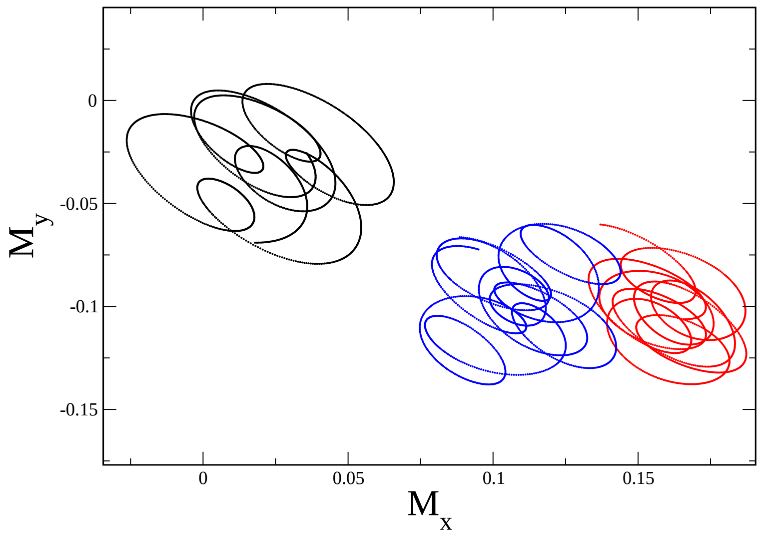

Figure 2. Three different trajectories of the total magnetization at the plane $M_z = 0$, for $R = 4$, $L = 4000$, $\delta = 10^{-3}$, $N = 64$, in the integrable case. Initially spins are chosen with random components on the unit sphere.

$$\frac{dM_x}{d\tau} = -\frac{3}{N}\sum_{i\neq k}\frac{1}{|r_{ik}|^3}\{S_i^z S_k^y + \epsilon N\,[S_k^x S_i^y \cos\phi_{ik} + (S_k^y S_i^y - S_k^z S_i^z)\sin\phi_{ik}]\},$$

and, in particular, for $\epsilon = 0$ we have :

$$\begin{aligned} \frac{dM_z}{d\tau} &= 0 \\ \frac{dM_y}{d\tau} &= \frac{1}{N}\sum_k \omega_k S_k^x \\ \frac{dM_x}{d\tau} &= -\frac{1}{N}\sum_k \omega_k S_k^y, \end{aligned} \tag{9}$$

having defined, the average "local" frequencies:

$$\omega_k = 3\sum_{i\neq k}\frac{1}{|r_{ik}|^3}S_i^z. \tag{10}$$

These equations describe a kind of rotation in the plane perpendicular to the z–magnetization (which is a constant of motion). Therefore one could expect that for $\epsilon N \ll 1$ a rotational-like motion about the z–axis persists, while M_z remains a quasi constant of motion. This is what can be observed for instance by a direct inspection of the trajectories of the macroscopic vector $\vec{M}$, in the plane x, y, see Fig. 2, where few selected trajectories has been iterated in time, for $\epsilon = 10^{-3}$ and $N = 64$. Quite

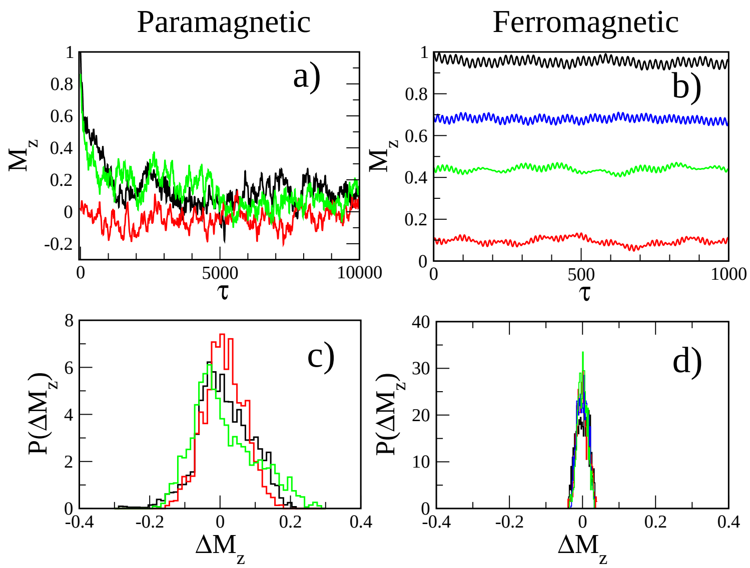

Figure 3. Data in this Figure refer to systems with $N = 64$ spins and a concentration $\delta = 10^{-3}$. The time behavior of the magnetization is shown, for different initial conditions, in the overcritical case $\epsilon = 0.125$ (a) ($L = 160, R = 20$) and in the undercritical case $\epsilon = 10^{-3}$ (b) ($L = 4000, R = 4$). In c) and d) the probability distribution functions for the fluctuations $\Delta M_z = (\langle M_z^2 \rangle - \langle M_z \rangle^2)^{1/2}$ around the equilibrium value is shown for the data given respectively in a) and b).

naturally, on increasing the perturbation strength ϵ, one could expect that the invariant tori $M_z = const$ will be broken, and, eventually, a stochastic motion of the macroscopic variable M_z will appear. In the next Section we will study the survival of invariant tori, under the dimensional perturbation $\epsilon N > 0$.

3. The chaotic–paramagnetic and the integrable–ferromagnetic phases

The dynamical behavior of the system can be characterized by a "regular region" $\epsilon N < 1$ in which the magnetization $M_z(\tau)$ is bounded in a small interval δM_z, while, for $\epsilon N > 1$, $M_z(\tau)$ quickly decays to zero. To be more precise, the transition across $(\epsilon N)_{cr} \simeq 1$ is smooth, namely there is a region of ϵN values in which the initial magnetization decay to some non zero constant when the time τ becomes large.

The critical value of the perturbation strength $(\epsilon N)_{cr} \simeq 1$ can be obtained with the following hand-waving argument. Let us divide the 3–D box in $n = L/R = 1/\epsilon$ small cubic boxes of side R. If the impurities concentration δ is sufficiently small in order to have only one spin inside each R-side box then the system is approximately one dimensional and M_z can be considered an approximate constant of motion. Otherwise, for large δ, the system behaves like a 3–D system and M_z can spread everywhere. In other words, in order to have less than one spin in each R^3 block one should have

$N/n < 1$, or $\epsilon N < (\epsilon N)_{cr} = 1$.

Moreover the study of the dynamics done in the previous Section suggests to take as a small parameter ϵN and to look for ferromagnetism when $\epsilon N < 1$. Note that this choice is also appropriate from the thermodynamic point of view since $\epsilon N = RN/L$ is an intensive parameter in the large N limit $L \to \infty, \quad N \to \infty, \quad N/L = const$, with R fixed.

An example is shown in Fig. 3, where the dynamics of magnetization has been plot in the overcritical case ($\epsilon N > 1$ Fig. 3a) and in the undercritical one ($\epsilon N < 1$ Fig. 3b). Different trajectories, corresponding to different initial conditions $M_z(0)$ have been shown in different colors. As one can see, in the "paramagnetic" phase ($\epsilon N > 1$) the magnetization first decays to zero and then it fluctuates randomly around zero. On the contrary, in the "ferromagnetic" phase ($\epsilon N < 1$), it shows a periodic behavior around the initial conditions.

This behavior is quite typical in the study of dynamical systems, where the increase of a suitable perturbative parameter is related to the breaking of invariant tori and to the emergence of chaotic motion [20, 21].

It is also remarkable to study the fluctuations around the asymptotic behavior: in the undercritical case (Fig. 3d) fluctuations are much smaller than in the overcritical case (Fig. 3c), roughly 10 times for this case, as can be seen comparing the width of the probability distribution functions in Fig. 3 c) and d).

The large fluctuations around the average values and in order to fit a possible experimental situation, suggest averaging over disorder, namely an ensemble of samples with different random configurations, initially magnetized along the z–axis.

The results for the ensemble average $\langle M_z \rangle$ are shown in Fig.4a, where the different behavior in the two "phases" $\epsilon N < 1$ and $\epsilon N > 1$ are reflected in an average magnetization not decreasing or decreasing to zero. Indeed, the average magnetization in the undercritical regime reaches some equilibrium value different from zero after some initial decay, while in the overcritical regime it goes to zero in an algebraic way.

Ensemble fluctuations at the equilibrium are independent of ϵN in the paramagnetic phase while in the ferromagnetic one are typically smaller and increasing as $\sqrt{\epsilon N}$. They are presented in Fig. 4b), where

$$\Delta M_z^{eq} = \lim_{\tau \to \infty} \Delta M_z(\tau)$$

has been shown as a function of ϵN. On the vertical axis we renormalize the asymptotic values by $\sqrt{(N)}$, to take into account fluctuations due to variation of the number of spins N. Each set of points on the plot corresponds to an ensemble of magnetized needles, with the same concentration δ and number of spins N (paramagnetic ions) and different aspect ratio ϵ, or same concentration and aspect ratio and different number of spins. It is quite remarkable that the critical value $\epsilon N \sim 1$, is well fitted by all different series, suggesting ϵN as a good scaling parameter for the macroscopic behavior.

Both the independence of the perturbation strength in the paramagnetic phase and the square root dependence on ϵN in the ferromagnetic phase can be understood on

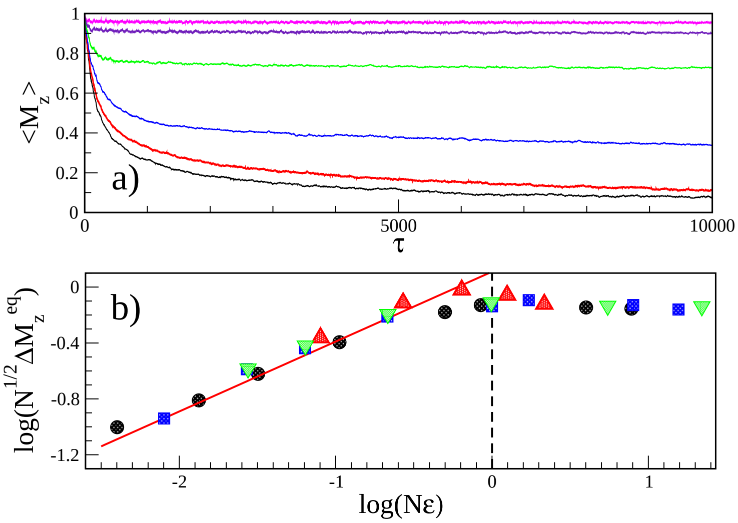

Figure 4. a) Time behavior of the average magnetization for different values of the aspect ratio $\epsilon = 1.25 \times 10^{-4}, 2.9 \times 10^{-4}, 9.8 \times 10^{-4}, 4.5 \times 10^{-3}, 2.5 \times 10^{-2}, 10^{-1}$ (from the upper to the lower), fixed number of spins $N = 220$ and fixed concentration $\delta = 10^{-3}$. The average is taken over 100 different random configurations. Initially we choose $S_i^z(0) = 1,\ i = 1, \ldots, N$. b) Dependence of the equilibrium value of fluctuations as a function of ϵN. Dashed vertical line indicates the critical value $\epsilon N = 1$. Red line indicates the dependence $\sqrt{N\epsilon}$. Different symbols stand for : $N = 40,\ \delta = 0.5\ 10^{-3}$ (circles), $N = 80,\ \delta = 10^{-3}$ (squares), $N = 220,\ \delta = 10^{-3}$ (triangles down), $\epsilon = 0.1,\ \delta = 10^{-2}$ (triangles up).

the basis of classical dynamical theory. Breaking invariant tori with a perturbation strength k corresponds to create stochastic layers between invariant tori whose size is proportional to $\sqrt{k}$[20, 21]. On the other side when the system is completely chaotic, since the variable M_z is bounded, it can only occupy all the allowable stochastic region, and a further increasing of perturbation strength can not modify this size.

Finally we point out that around $\epsilon N = 1$ we can expect a transition from a ferromagnetic ground state to an antiferromagnetic ground state. Indeed for $\epsilon N \ll 1$ the system is close to a $1D$ arrangement of dipoles, so that the ground state will be ferromagnetic as pointed out in the previous Section. On the other side for $\epsilon N \gg 1$ the system is close to a $3D$ arrangement of dipoles. In this case, for a simple cubic lattice, it can be shown [19] that the ground state is antiferromagnetic. Some numerical simulations we did confirm this conjecture, but work is still in progress and will be presented in a future publication.

4. Demagnetization time

As we have seen in the previous Section, the system dynamics can be described by the parameter, ϵN, characterizing two different dynamical phases, and describing how much

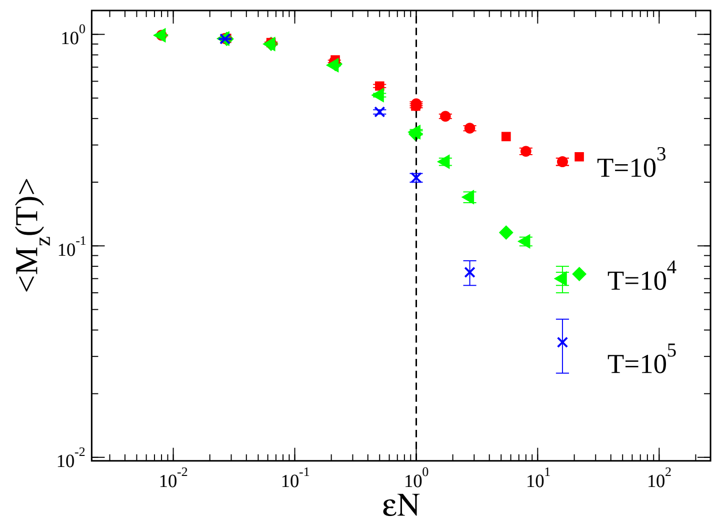

Figure 5. Average Magnetization at time T as a function of ϵN. For the same ϵN we show the average magnetization after different simulation times, T. Data refer to the case $\delta = 10^{-3}$. Different number of dipoles are shown: $N = 220$ (circles for $T = 10^3$ and lozanges for $T = 10^4$) and $N = 80$ (squares for $T = 10^3$, left trangles for $T = 10^4$ and crosses for $T = 10^5$).

one–dimensional a system is. In this Section we will show that ϵN is also a good (and intensive) scaling parameter for the macroscopic properties of the quasi 1–D system.

In order to prove numerically such argument we need to find two physical observables that can describe the paramagnetic and the ferromagnetic phase and to study their dependence on the parameter ϵN. To this end, let us introduce, on the paramagnetic side, the demagnetization time $\tau_{1/2}$, defined as the time at which the average magnetization decay to one half of its initial value:

$$\langle M_z(\tau_{1/2})\rangle = (1/2)\langle M_z(0)\rangle.$$

In the same way, on the ferromagnetic side, we introduce the "remnant magnetization" M_r as the magnetization left when $\tau \to \infty$,

$$M_r = \lim_{\tau\to\infty} \langle M_z(\tau)\rangle.$$

Let us stress that both quantities are physically sound, in the sense that they are directly and easily measurable.

While it is clear that, even if both quantities can be defined only in their respective phases, they can give useful information when extended to the other phases. For instance, in the paramagnetic phase the average magnetization will depend strongly on the simulation time (T), while in the ferromagnetic phase the demagnetization time is typically infinite.‡ The dependence of the average magnetization at fixed time T can

‡ In this case we underestimate its value putting the maximum dimensionless simulation time, which is 10^4.

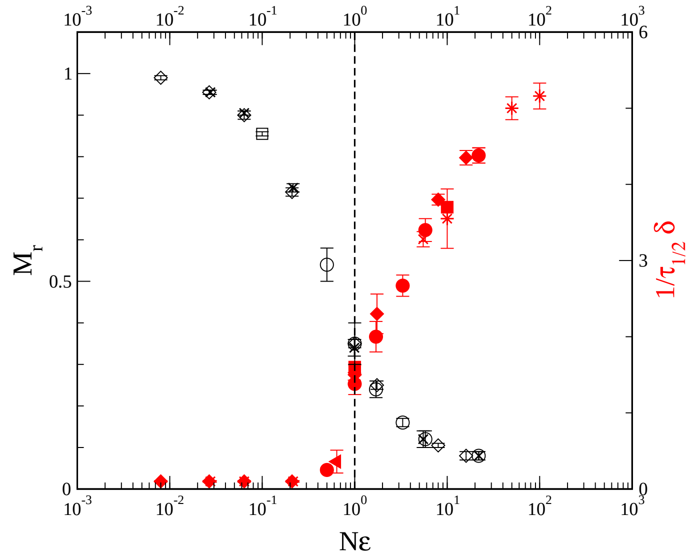

Figure 6. Left axis : remnant magnetization M_r *vs* the parameter ϵN indicated with open symbols. Rigth axis : inverse demagnetization time $\tau_{1/2}$ rescaled by the concentration δ, *vs* the parameter ϵN indicated with full symbols. List of symbols: circles ($\epsilon = 0.1, \delta = 10^{-3}$), lozanges ($N = 80, \delta = 0.01$), crosses ($N = 220, \delta = 10^{-3}$), squares ($N = 100, R = 4$), asterisks ($N = 500, R = 20$), left triangles ($\epsilon = 0.01, \delta = 0.01$). Initially we choose $S_i^z(0) = 1$, $i = 1, \ldots, N$. In this Figure an ensemble of 100 different configurations has been considered. Each member of the ensemble has been integrated for 10^4 dimensionless time units.

give important information on the ferromagnetic-paramagnetic transition. Indeed we can expect a weak dependence of the average magnetization on the simulation time T, in the ferromagnetic region $\epsilon N < 1$, since the presence of quasi-constant of motions freezes the magnetization, while, in the paramagnetic phase, the average magnetization at time T, $< M_z(T) >$, goes to zero as the time grows. This fact is clearly shown in Fig. (5), where the average magnetization $< M_z(T) >$ is plotted *vs* ϵN for different times T. As we can see from Fig. (5), as we increase the time T the average magnetization remains almost constant in the ferromagnetic side ($\epsilon N < 1$), while it goes to zero in the paramagnetic side ($\epsilon N > 1$), thus demonstrating a clear signature of the dimensional transition discussed above.

Since $\epsilon N = \delta R^3$, one can essentially consider different ways to approach the critical point $\epsilon N = \delta R^3 \simeq 1$, keeping fixed one of the four quantities ϵ, N, δ, R and varying correspondingly the others. This is exactly what we did in Fig. 6, where we show, on the same plot, the remnant magnetization M_r (open symbols and vertical axis to the left), and the inverse demagnetization time, $\tau_{1/2}\delta$, rescaled by the concentration δ (vertical axis to the right, full symbols) both as a function of the parameter ϵN. Reserving later on the discussion about the time-rescaling with δ, let us observe two relevant features. The first one is the presence of a change of curvature of both curves

on approaching the critical border $\epsilon N = \delta R^3 \simeq 1$. The second is the scaling of all points in the two curves (one for $\tau_{1/2}\delta$, the other for the remnant magnetization M_r.)

As for the rescaling of the time, let us observe that, due to the particular quasi 1–D geometry, and to the low concentration $\delta \ll 1$, closest dipoles give the major contribution to the energy. For instance, the configuration with all spins aligned along the z–axis will have an energy,

$$E' \propto \sum_{\langle i,j \rangle} \frac{1}{|r_{ij}|^3}, \tag{11}$$

where the sum is taken over N couples $\langle i,j \rangle$ of neighbor dipoles. In other words $E' \sim N/\langle d \rangle^3 \sim N\delta$, where $\langle d \rangle$ is the average distance between two dipoles.

On the other hand the Landau-Lifshitz-Gilbert equations of motion are invariant under a simultaneous scaling of time and energy $\tau' = \tau/\delta$ and $E' = E\delta$ so that we will expect $\tau \propto 1/\delta$. This simple relation has been verified considering a system with the same aspect ratio ϵ, and the same number of particles N (so to have the same value of ϵN) and changing the concentration δ over 3 orders of magnitude. Results are presented in Fig. 7a, where $\tau_{1/2}$ has been shown *vs* δ. To guide the eye a dashed line indicating the inverse proportionality has been superimposed. As one can see, looking at the last point to the right side of Fig. 7a, this relation does not hold true for concentrations $\delta \simeq 1$, where the nearest neighbor approximation (11) fails.

The "scale invariance" is even more evident if the average magnetization is considered as a function of the rescaled time $\tau\delta$, shown in Fig. 7b, for the same cases belonging to the straight line shown in Fig. 7a).

At last, we investigate the system behavior on approaching the large N limit. First of all let us observe that the scaling variable $\epsilon N = RN/L$ is well defined in the large N limit $N, L \to \infty$, $N/L = const$ and R fixed.§

In order to do that we take into account different systems with fixed concentration δ and radius R and increasing lenght L and number of particles N: both in the ferromagnetic phase $\epsilon N < 1$ (Fig. 8a), and in the paramagnetic one $\epsilon N > 1$ (Fig. 8b) , the average magnetization is independent on the number of particles N.

5. Conclusions

In this paper the microcanonical dynamics of a system of random dipoles, interacting with a pure dipole-dipole interaction has been considered. We have shown that a dimensional "phase" transition, correspondent to a transition from regular (ferromagnetic) to stochastic (paramagnetic) regime occurs, in the microcanonical ensemble, for low concentration δ. Such transition is characterized from the dynamical point of of view by a different behavior of the fluctuations of the average magnetization, and from the physical point of view respectively by a zero remnant magnetization, $M_r = \lim_{\tau\to\infty}\langle M_z(\tau)\rangle$, and finite decay rates, $\propto 1/\tau_{1/2}\delta$ (Paramagnetic Phase) or zero

§ We thank the Referee for this remark.

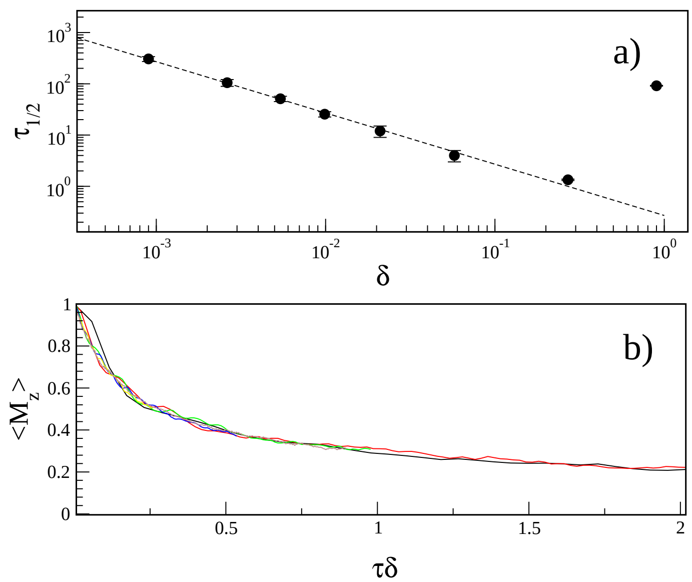

Figure 7. a) Dependence of the average demagnetization time $\tau_{1/2}$ as a function of concentration δ, for systems with $\epsilon = 0.1$ and $N = 72$. The average has been taken over an ensemble of 100 different samples. Initially all samples have all spins aligned along the z–axis : $S_i^z(0) = 1, \quad i = 1, \ldots N$. Dashed line represents $\tau_{1/2} \propto 1/\delta$. b) Average magnetization $\langle M_z(\tau)\rangle$ as a function of the rescaled time $\tau\delta$ for different concentration δ and fixed $\epsilon = 0.1$, and $N = 72$ as in a).

decay rates and finite remnant magnetization (Ferromagnetic Phase). We showed that this dimensional transition occurs when the intensive parameter $\epsilon N = 1$, where ϵ is the aspect ratio and N is the number of dipoles. For instance in an experimental situation if we have a non magnetic substrate with $R = 1.6$ nm and $L = 1.6$ μm, with lattice size ~ 40.4 nm, we expect a dimensional transition for $\delta = 0.15\%$. We also conjectured that in correspondence to this transition, the ground state changes from ferromagnetic to antiferromagnetic.

In the future we would like to investigate dilute dipole systems in the canonical ensemble, that is letting the system be in contact with a thermal bath. Our analysis in the microcanonical ensemble indicated that the behavior of very dilute dipoles in a needle geometry is very similar to a 1–D arrays of dipoles. In the 1–D case dipole interaction induces a ferromagnetic ground state, and, due to its anisotropy, to a breaking of ergodicity [3]. As shown in Ref. [23], the ergodicity breaking threshold can induce very large demagnetization times thus producing ferromagnetic behavior in finite samples. Thus, even if one would expect that invariant tori will be destroyed under a suitable thermal perturbation, the question on the demagnetization times in presence of temperature and on the relevance of the ergodicity breaking is still open.

The ergodicity breaking found in Ref. [3] considers the total magnetization as an order parameter. On the other hand, different order parameters can be defined in dipole

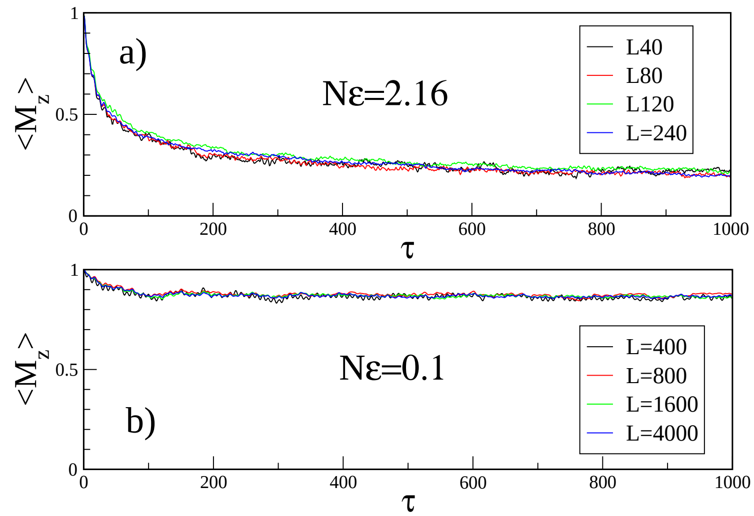

Figure 8. Average magnetization $\langle M_z \rangle$ *vs* the dimensionless time τ, for different sample lengths L, as indicated in the legend, and different number of spins N, at fixed density ($N/L = 0.36$) for the paramagnetic phase $N\epsilon = 2.16$ (a), and for the ferromagnetic one $\epsilon N = 0.1$ (b). In a) is $R = 6$, $\delta = 0.01$, while in b) is $R = 4$ and $\delta = 0.0015625$. Initially we choose $S_i^z(0) = 1$, $i = 1, \ldots, N$. An ensemble of 100 different configurations has been considered.

systems, depending on the ground state configuration, for instance an anti-ferromagnetic order parameter or a spin glass order parameter. Therefore, it would be interesting to investigate the existence of an ergodicity breaking energy threshold with respect to different order parameters.

In conclusion dipole-dipole interacting spin systems offer a realistic playground to analyze many properties of magnetic systems which challenge our comprehension.

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